APPENDIX E
IMPLEMENTATION OF SLANT STACKS

I shall now derive several formulations of global and local slant stacks. First I shall derive a global, frequency-domain version with sufficient dips for inversion. Then I shall discuss how to avoid possible artifacts from the frequency-domain interpolation.

E.1. Frequency-domain slant stacks

For sections of many traces a frequency domain slant stack becomes less expensive than the space-time version. Lateral wrap-around may be avoided with the proper interpolator. Define slant stacks by the inverse transformation.

$$d(x,t) = \mathbf{F}\mathbf{d}' = \int d'(p, \tau = t - px)\, dp \tag{E.1}$$

Thus, a single point in the slant stack domain $d'(p,\tau) = \delta(p - p_0)\,\delta(\tau - \tau_0)$ will map to a line in the spatial domain. Two points map to two additively superimposed lines. Let the variables (x,t,p,τ) have the Fourier duals of (k,s,q,ν). Fourier transforming to $d(x,s)$ and $d'(q,\nu)$ yields a simplified relation.

$$d(x,t) = \int\int\int e^{i2\pi\nu(t - px)}\, e^{i2\pi qp}\, d'(q,\nu)\, dp\, dq\, d\nu \tag{E.2}$$

$$= \int\int\int e^{i2\pi p(q - \nu x)}\, e^{i2\pi\nu t}\, d'(q,\nu)\, dp\, dq\, d\nu = \int e^{i2\pi\nu t}\, d'(q = \nu x, \nu)\, d\nu$$

$$\rightarrow\ d(x,s) = d'(q = sx, \nu = s) \tag{E.3}$$

This transformation may be performed as a frequency domain stretch. Unfortunately, in the forward direction a rectangle is mapped to a triangle. Dips not in the range of calculated p's must be aliased. The adjoint transformation, however, implicitly zeros those dips omitted from $\mathbf{d}'$.

$$d'(p,\tau) = \mathbf{F}^*\mathbf{d} = \int d(x, t = \tau + px)\, dx \tag{E.4}$$

$$\rightarrow d'(p,\nu) = d(k = -\nu p, s = \nu) \tag{E.5}$$

Use (E.3) by first performing (E.5), then following with an equivalent of the *rho* filter of Radon transforms. This filter is the inverse $(\mathbf{F}^*\mathbf{F})^{-1}$.

$$\tilde{d}'(q,\nu) = \mathbf{F}^*\mathbf{F}\mathbf{d}' = \int\int e^{i2\pi(\nu p)x}\, e^{-i2\pi qp}\, d'(q = \nu x, \nu)\, dx\, dp$$

$$= \int \delta(\nu x - q)\, d'(q = \nu x, \nu)\, dx$$

$$= \int \frac{1}{|\nu|}\, \delta\!\left(x - \frac{q}{\nu}\right) d'(q = \nu x, \nu)\, dx = \frac{1}{|\nu|}\, d'(q,\nu)\, dx \tag{E.6}$$

$$\rightarrow\ d'(p,\nu) = \mathbf{F}^{-1}\mathbf{d} = (\mathbf{F}^*\mathbf{F})^{-1}\mathbf{F}^* = |\nu|\ d(k = -\nu p, s = \nu) \tag{E.7}$$

Forward transform with (E.7) to avoid aliasing dips and inverse transform with (E.3).

p values should be sampled well enough to avoid aliasing frequencies in traces at high x. Equivalently, the sampling in (E.7) should satisfy

$$\Delta p \cdot s_{\max} < \Delta k$$

The discrete equivalent becomes

$$\Delta p < \frac{2 \cdot \Delta t}{N_x \cdot \Delta x} \tag{E.8}$$

where N_x is the number of x samples.

E.2. Frequency-domain interpolation

The discrete frequency-domain stretch over k requires an interpolation operator to implicitly zero space-domain replications responsible for "wrap-around." Interpolation operators are equivalent to convolutions. Convolving by a function over k is equivalent to multiplying by its Fourier transform over x. $d(x,t)$ should be multiplied by a rectangle function that drops to zero before the first trace and after the last. Let us derive a general interpolator and allow arbitrary implicit windowing functions in the other domain. A local slant stack should use a Gaussian window. Stolt migration requires a similar stretch over temporal frequencies; a rectangular window should adjust to the location of zero time.

Define three parameters: x_z, the coordinate to be newly assigned the value of zero in the x domain; x_c, the center (in new coordinates) of the function to multiply the x domain; X_w, the width of the function to multiply the x domain. Assume the function $W(x)$ to be symmetric about zero. Windowing $d(x)$ with $W(x)$ gives the transform

$$W\left(\frac{x - x_c}{X_w}\right) d(x) \text{ transforms to } \int e^{-i 2\pi x_c (k - k')} X_w \; W' [X_w (k - k')] \, e^{i 2\pi x_z k'} d'(k') \, dk' \tag{E.9}$$

(Dimensions not being stretched are suppressed.) $W'(s)$ is the Fourier transform of $W(x)$. Interpolation merely adapts (E.9) to the discrete case. Let n_z, n_c, and N_w be the important parameters in samples.

$$d'_{n+\delta n} = \sum_{m=0}^{N_x - 1} \frac{N_w}{N_x} W' \left(\frac{(n + \delta n - m) N_w}{N_x} \right) e^{-i 2\pi n_c (n + \delta n - m)/N_x} \; e^{i 2\pi n_z m / N_x} \; d_m' \tag{E.10}$$

For a global slant stack and for Stolt migration with the surface at the first sample use $n_z = 0$, $n_c = N_x/2$, and $N_w = N_x$. Transform the rectangle function of equation (D.7) into the sinc function. The following simplification occurs (cf. Rosenbaum).

$$d_{n+\delta n} = \frac{1}{\pi} e^{-i\pi\delta n} \sin \pi\delta n \sum_{m=0}^{N_x-1} d_m / (n+\delta n-m) \tag{E.11}$$

(E.11) may be tapered to few terms.

When the windowing function $W(x)$ becomes a Gaussian, one may minimize the number of terms in the interpolation operator. Let $W(x) = \exp(-\pi x^2)$ so that X_w governs the distance between half-amplitude points $[W(0.5) \approx 0.5]$. Then $W'(s) = \exp(-\pi s^2)$. Preserve all terms in (E.10) with coefficient magnitudes greater than 0.01. Then

$$| n+\delta n-m | < 1.2 \frac{N_x}{N_w} .$$

The number of terms in the interpolator should be greater than approximately $3 \cdot N_x / N_w$.

Note that smaller N_w's allow larger Δp's. When $n_c = 0$, N_w may replace N_x in (E.8).

APPENDIX G

EXTRACTING DIFFRACTIONS WITH SPATIALLY VARIABLE VELOCITIES

To apply the extractions of diffractions in section 2.3 to a larger window of data, we should like to vary velocities vertically and laterally. Those results could be applied directly by partitioning migrated sections and extracting diffractions independently in each. This approach, however, requires troublesome data organization and has particular problems at partition boundaries. Instead let us assume smooth polynomial variations in rock velocities. (Other smooth modulations, such as low frequency sines and cosines, also apply.)

To allow polynomial variations of diffraction velocity with depth, rewrite (2.4) and (2.5) to read

$$\mathbf{e}_v^n \equiv \mathbf{F}_v^{-1} \cdot \text{Gain by } z^n \cdot \text{Extract}\{\mathbf{F}_v \cdot \mathbf{data}\} \tag{G.1}$$

$$\hat{\mathbf{s}} = \sum_n \sum_v a_v^n \, \mathbf{e}_v^n \; ; \;\; \min_{\mathbf{a}} \, || \, \hat{\mathbf{s}} - \mathbf{data} \, ||^{\,2} \;\; \rightarrow \;\; \sum_m \sum_w a_w^m (\mathbf{e}_v^n \cdot \mathbf{e}_w^m) = \mathbf{e}_v^n \cdot \mathbf{data}. \tag{G.2}$$

An additional gain multiplies by powers of the depth coordinate z. Lateral changes could be added with lateral gains. The least-squares solution may now smoothly change the weighting of extracted events spatially. Solving (G.2) requires inverting a symmetric matrix of an order equal to the number of velocities times the number of gains. Unfortunately, each additional power in the polynomial variation should require another inverse transform, $\mathbf{F}_v^{-1}$, for each velocity.

If diffractions with substantially different velocities do not overlap in the time section, however, then the gain and the inverse transform should commute well.

$$\mathbf{e}_v^n \equiv \text{Gain by } t^n \cdot \mathbf{F}_v^{-1} \cdot \text{Extract}\{\mathbf{F}_v \cdot \mathbf{data}\} \tag{G.3}$$

Gains over the time coordinate t may be made implicitly in the scalar products. Thus, the additional inverse transforms may be avoided, and variable velocities added cheaply, by calculation of additional scalar products.

Since the superposition allows spatially variable velocities, all migrations and diffractions may use constant velocities. To estimate the spatially variable velocities, migrate $\hat{\mathbf{s}}$ over the proper range of velocities and evaluate the focusing measure in tapered windows.