APPENDIX F

FREQUENCY-DOMAIN MIGRATION

For convenience I briefly derive the dispersion relations used for migration in this paper. Constant velocity formulations will suffice for the applications of Chapter 2; least-squares superpositions can be spatially variable. Stream-lined Stolt (f-k) or Gazdag (phase shift) algorithms are the most efficient for multiple constant-velocity migrations.

Let us begin in every case with the double-square root equation (DSR). Assume data are recorded as a function of (s, g, t), which have the Fourier duals (k_s, k_g, ω). s is the horizontal coordinate of the shot, g of the geophone; t is the arrival time. z is the depth of an imaged reflector, and k_z its dual.

$$k_z = \frac{\omega}{v} [\sqrt{1 - S^2} + \sqrt{1 - G^2}] \tag{F.1}$$

$$\text{where } S = \frac{vk_s}{\omega} \ , \ G = \frac{vk_g}{\omega}$$

See Claerbout (1984) for a derivation and justification of this relation. In short, a single square root derives from the scalar wave equation: $\omega^2 = {k_z}^2 + {k_x}^2$; reciprocity allows shots to be downward continued just as geophones. No one uses this relation directly. Nevertheless, in theory one could migrate by mapping the data from (k_s, k_g, ω) to (k_s, k_g, k_z) and imaging at $(s = g, z)$. Ottolini, 1982 provides some insightful use of this relation in various coordinate systems.

Often, midpoint-offset coordinates are more convenient: $y = (g + s)/2$, $h = (g - s)/2$.

$$k_z = \frac{\omega}{v} [\sqrt{1 - (Y + H)^2} + \sqrt{1 - (Y - H)^2}] \tag{F.2}$$

$$\text{where } Y = \frac{vk_y}{2\omega} \ , \ H = \frac{vk_h}{2\omega}$$

A stacked section is a sum of constant offset sections stretched (by normal moveout and perhaps dip moveout) to resemble the zero offset. The data then are a function of (y, t) and supposedly invariant over h, thus $k_h \equiv 0$. Migration requires mapping from (k_y, ω) to (k_y, k_z) with

$$k_z = \frac{2\omega}{v} \sqrt{1 - Y^2} \ . \tag{F.3}$$

Stolt and Gazdag give two widely used algorithms for this mapping, the commonest migration. Stolt's is fastest, but a Gazdag's will treat depth variable velocities accurately. Use Stolt's method to estimate velocities from diffraction events. These algorithms also apply to the next dispersion relation.

Wave-equation stacks of common-midpoint (common-depth-point) gathers should recognize dips in the orthogonal direction, along midpoint (see section 2.4.2.). Begin with a narrow cube of seismic data, a function of (y, h, t), that contains 4 to 8 adjacent midpoint gathers. Decompose dipping events over y and t with equation (2.7) for data as a function of (p_y, y_c, t, h), where y_c is the midpoint of the central gather. Signal extraction should follow to discriminate against noise and artifacts from truncation of the data. A given common-midpoint gather contains only events with a known dip along midpoint $p_y = k_y / \omega$. Map (p_y, y_c, ω, k_h) to (p_y, y_c, k_z, k_h) with

$$k_z = \frac{\omega}{v} \left[\sqrt{1 - (H + vp_y/2)^2} + \sqrt{1 - (H - vp_y/2)^2}\,\right] . \tag{F.4}$$

Migrating with (F.4) as F_v in (G.1) and (G.2) of Appendix G will provide a depth variable extraction of events containing velocity information. The focusing measure will then identify the best depth variable velocities.