$$\mathbf{X}_S$$

$\mathbf{X}_r$

$$0 \leq t \leq T$$

$$c(\mathbf{x})$$

$$\rho(\mathbf{x})$$

$P_s(\mathbf{x}, t)$

$$\left[\frac{1}{c(\mathbf{x})^2 \rho(\mathbf{x})} \frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla} \cdot \left(\frac{1}{\rho(\mathbf{x})} \boldsymbol{\nabla} \right) \right] P_s(\mathbf{x}, t) = f_s(t) \delta(\mathbf{x} - \mathbf{x}_s) \delta(t),$$

$$f_s(t) = \boldsymbol{\nabla}\cdot[\mathbf{f}(t)/\rho(\mathbf{x}_s)],$$

$$\mathbf{f}(t)$$

$$P_s(\mathbf{x},t)\big|_{z\leq 0} = 0, \quad P_s(\mathbf{x},t)\big|_{t\leq 0} = 0, \text{ and } \frac{\partial^2}{\partial t^2}P_s(\mathbf{x},t)\bigg|_{t\leq 0} = 0,$$

$$z = x_1$$

X

P

$\partial\Omega$

V_{max}

T

the surface $\partial\Omega$

=

$$\{\mathbf{x} \mid z = 0 \quad \text{or} \quad \| \mathbf{x} \| = TV_{max}\},$$

for the region Ω

$$\{\mathbf{x} \mid z \geq 0 \ \text{ and } \ \| \mathbf{x} \| \leq TV_{max}\}.$$

$$P_s(\mathbf{x},t)\big|_{\mathbf{x}\in\partial\Omega} = 0.$$

$$\int_0^T \int_\Omega \left\{ \left[\frac{1}{c(\mathbf{x})^2 \rho(\mathbf{x})} \frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla} \cdot \left(\frac{1}{\rho(\mathbf{x})} \boldsymbol{\nabla} \right) \right] P_s(\mathbf{x}, t) - f_s(t) \delta(\mathbf{x} - \mathbf{x}_s) \delta(t) \right\} q(\mathbf{x}, t) \, d\mathbf{x} \, dt = 0,$$

H^2

$$q(\mathbf{x}, t)$$

q

$D_{rs}(t)$

ρ

C

$f_s(t)$

$$J[\rho, c, f_s]$$

$$\frac{1}{2}\sum_{r,s}\int_0^T \left[D_{rs}(t) - \frac{1}{\rho(\mathbf{x}_r)}\frac{\partial P(\mathbf{x}_r, t)}{\partial z}\right]^2 \; w(t)dt$$

$$\frac{1}{2}\sum_{r,s}\int_0^T\int_\Omega \left[D_{rs}(t) - \frac{1}{\rho(\mathbf{x})}\frac{\partial P_s(\mathbf{x},t)}{\partial z}\right]^2 \delta(\mathbf{x}-\mathbf{x}_r)\ d\mathbf{x}\, w(t)\, dt.$$

$$w(t)$$

S

$$w(t)$$

$$\int_0^T \int_\Omega q(\mathbf{x}, t) L_1[\mathbf{m}(\mathbf{x}, t)] P_s(\mathbf{x}, t) \; d\mathbf{x} \, dt = 0, \;\; \forall q(\mathbf{x}, t).$$

where $\mathbf{m}(\mathbf{x}, t) = [c(\mathbf{x}), \rho(\mathbf{x}), f_s]$.

$\mathbf{m}(\mathbf{x}, t)$

L_1

$$1/c^2\rho$$

$1/\rho$

f_s

m

$$J[\mathbf{m}(\mathbf{x},t)] = \frac{1}{2}\sum_{r,s}\int_0^T\int_\Omega \left\{D_{rs}(t) - L_2[\mathbf{m}(\mathbf{x},t),t]P_s(\mathbf{x},t)\right\}^2 \delta(\mathbf{x}-\mathbf{x}_r)\; w(t)\; d\mathbf{x}\,dt.$$

$\delta \mathbf{m}$

δP_s

$$\int_0^T \int_\Omega q L_1 \delta P_s \, d\mathbf{x} \, dt = -\int_0^T \int_\Omega q \boldsymbol{\nabla}_{\mathrm{m}} (L_1 P) \cdot \delta \mathbf{m} \, d\mathbf{x} \, dt, \ \forall q.$$

$$\nabla_{\mathrm{m}} G(\mathbf{m}) \cdot \delta \mathbf{m} = \frac{d}{d\epsilon} G(\mathbf{m} + \epsilon \delta \mathbf{m}) \bigg|_{\epsilon=0}, \text{ where } G(\mathbf{m}) = L_1 P,$$

$$\boldsymbol{\nabla}_{\mathrm{m}}(L_1 P)$$

a

b

L^*

L

$$< a|Lb >_a = < L^* a|b >_b$$

$$< q_1 | q_2 >_q$$

$$\int_0^T \int_\Omega q_1(\mathbf{x},t) q_2(\mathbf{x},t) \; d\mathbf{x} \, dt,$$

$$< P_1|P_2 >_P$$

$$\int_0^T \int_\Omega P_1(\mathbf{x}, t) P_2(\mathbf{x}, t) \; d\mathbf{x} \, dt, \text{ and}$$

$$< m_1 | m_2 >_m$$

$$\int_{\Omega} m_1(\mathbf{x}) m_2(\mathbf{x}) \, d\mathbf{x}.$$

$$\int_0^T \int_\Omega \delta P_s L_1^* q \; d\mathbf{x} \, dt = -\int_\Omega \delta \mathbf{m} \int_0^T [\boldsymbol{\nabla}_{\mathrm{m}}(L_1 P)]^* q \; dt \; d\mathbf{x}, \;\; \forall q.$$

L_1^*

J

L_2

$$-\delta J$$

$$\sum_{r,s}\int_0^T\int_\Omega \delta(\mathbf{x}-\mathbf{x}_r)(D_{rs}(t)-L_2P_s)L_2\delta P_s\ w\quad d\mathbf{x}\,dt$$

+

$$\sum_{r,s}\int_0^T\int_\Omega \delta(\mathbf{x}-\mathbf{x}_r)(D_{rs}(t)-L_2P_s)\boldsymbol{\nabla}_{\mathrm{m}}(L_2P_s)\cdot\delta\mathbf{m}\ w \quad d\mathbf{x}\,dt$$

$$\sum_{r,s}\int_0^T\int_\Omega \delta P_s L_2^*[\delta(\mathbf{x}-\mathbf{x}_r)(D_{rs}(t)-L_2 P_s)\ w]\ d\mathbf{x}\,dt$$

$$\sum_{r,s}\int_{\Omega}\delta\mathbf{m}\cdot\int_0^T[\boldsymbol{\nabla}_{\mathrm{m}}(L_2P_s)]^*[\delta(\mathbf{x}-\mathbf{x}_r)(D_{rs}(t)-L_2P_s)\ w]dt\ d\mathbf{x}.$$

$$\delta P$$

$$\int_0^T \int_\Omega \delta P L_1^* q \; d\mathbf{x}\, dt = \sum_{r,s} \int_0^T \int_\Omega \delta P L_2^* [\delta(\mathbf{x} - \mathbf{x}_r)(D_{rs}(t) - L_2 P) \; w] \; d\mathbf{x}\, dt.$$

$$L_1^* q = \sum_{r,s} L_2^* [\delta(\mathbf{x} - \mathbf{x}_r)(D_{rs}(t) - L_2 P) w].$$

$$\delta J$$

$$\int_\Omega \delta \mathbf{m} \int_0^T [\boldsymbol{\nabla}_{\mathrm{m}}(L_1 P)]^* q \ dt \ d\mathbf{x}$$

—

$$\sum_{r,s} \int_{\Omega} \delta \mathbf{m} \int_0^T [\boldsymbol{\nabla}_{\mathrm{m}}(L_2 P)]^* [\delta(\mathbf{x} - \mathbf{x}_r)(D_{rs}(t) - L_2 P) w] \; dt \; d\mathbf{x}.$$

$$\mathbf{m}(\mathbf{x})$$

$$\boldsymbol{\nabla}_{\mathrm{m}} J[\mathbf{m}(\mathbf{x})] = \left.\frac{\delta J}{\delta \mathbf{m}}\right|_{\mathbf{m}(\mathbf{x})}$$

$$\int_0^T [\nabla_{\mathrm{m}}(L_1 P)]^* q \; dt$$

$$\sum_{r,s}\int_0^T [\boldsymbol{\nabla}_{\mathrm{m}}(L_2P)]^*[\delta(\mathbf{x}-\mathbf{x}_r)(D_{rs}(t)-L_2P)w]\ dt.$$

$$\left\{ \mathbf{X}_r \right\}$$

$$\min_{\alpha} J[\mathbf{m}_n + \alpha \boldsymbol{\nabla}_{\mathrm{m}} J(\mathbf{m}_n)], \text{ and } \mathbf{m}_{n+1} = \mathbf{m}_n + \alpha \boldsymbol{\nabla}_{\mathrm{m}} J(\mathbf{m}_n).$$

$$\mathbf{m}_{n+1}$$

$$-\delta J[\rho, c, f_s]$$

$$-\sum_{r,s}\int_0^T\int_\Omega \delta(\mathbf{x}-\mathbf{x}_r)\left[D_{rs}(t)-\frac{1}{\rho(\mathbf{x})}\frac{\partial P_s(\mathbf{x},t)}{\partial z}\right]\frac{1}{\rho(\mathbf{x})}\frac{\partial}{\partial z}\delta P_s(\mathbf{x},t)\;d\mathbf{x}\,w(t)\,dt$$

$$\sum_{r,s}\int_0^T\int_\Omega \delta(\mathbf{x}-\mathbf{x}_r)\left[D_{rs}(t)-\frac{1}{\rho(\mathbf{x})}\frac{\partial P_s(\mathbf{x},t)}{\partial z}\right]\frac{\partial P_s(\mathbf{x},t)}{\partial z}\frac{\delta\rho(\mathbf{x})}{\rho(\mathbf{x})^2}\;d\mathbf{x}\,w(t)\,dt.$$

Z

$$-\delta J[\rho, c, f_s] = \sum_{r,s} \int_0^T \int_\Omega \delta P_s(\mathbf{x}, t) \frac{\partial}{\partial z} \left\{ \delta(\mathbf{x} - \mathbf{x}_r) \frac{1}{\rho(\mathbf{x})} \left[D_{rs}(t) - \frac{1}{\rho(\mathbf{x})} \frac{\partial P_s(\mathbf{x}, t)}{\partial z} \right] \right\} \, d\mathbf{x} \, w(t) \, dt$$

$$+\sum_{r,s}\int_0^T\int_\Omega \delta\rho(\mathbf{x})\left\{\delta(\mathbf{x}-\mathbf{x}_r)\frac{1}{\rho(\mathbf{x})^2}\left[D_{rs}(t)-\frac{1}{\rho(\mathbf{x})}\frac{\partial P_s(\mathbf{x},t)}{\partial z}\right]\frac{\partial P_s(\mathbf{x},t)}{\partial z}\right\}\,d\mathbf{x}\,w(t)\,dt.$$

$$\delta P_s(\mathbf{x}, t)$$

$$\int_0^T \int_\Omega \left[\frac{1}{c(\mathbf{x})^2 \rho(\mathbf{x})} \frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla} \cdot \left(\frac{1}{\rho(\mathbf{x})} \boldsymbol{\nabla} \right) \right] \; \delta P_s(\mathbf{x}, t) \; d\mathbf{x} \, dt$$

$$\int_0^T \int_\Omega q(\mathbf{x}, t) \left[\frac{1}{c(\mathbf{x})^3 \rho(\mathbf{x})} \frac{\partial^2 P_s(\mathbf{x}, t)}{\partial t^2} \right] \delta c(\mathbf{x}) \ d\mathbf{x} \, dt$$

$$\int_0^T\!\!\int_\Omega q(\mathbf{x},t)\left\{\frac{1}{c(\mathbf{x})^2\rho(\mathbf{x})^2}\frac{\partial^2 P_s(\mathbf{x},t)}{\partial t^2} - \boldsymbol{\nabla}\cdot\left[\frac{1}{\rho(\mathbf{x})^2}\boldsymbol{\nabla} P_s(\mathbf{x},t)\right]\right\}\delta\rho(\mathbf{x})\;d\mathbf{x}\,dt$$

$$\int_0^T \int_\Omega q(\mathbf{x}, t)\delta(\mathbf{x})\delta f_s(t) \; d\mathbf{x} \, dt.$$

$$\int_{\Omega} a \boldsymbol{\nabla} \cdot \mathbf{b} \; d\mathbf{x}$$

$$\int_\Omega \nabla\cdot(a\mathbf{b})\ d\mathbf{x} - \int_\Omega \mathbf{b}\cdot\nabla a\ d\mathbf{x}$$

$$\int_{\partial\Omega} \hat{\mathbf{n}} \cdot \mathbf{b}\, a \, d\sigma - \int_{\Omega} \mathbf{b} \cdot \boldsymbol{\nabla} a \; d\mathbf{x}.$$

b

$\hat{\mathbf{n}}$

Ω

$$\int_{\Omega} q(\mathbf{x},t) \frac{1}{c(\mathbf{x})^2 \rho(\mathbf{x})} \frac{\partial}{\partial t} \delta P_s(\mathbf{x},t) \; d\mathbf{x} \bigg|_{t=T} - \int_{\Omega} \frac{\partial q(\mathbf{x},t)}{\partial t} \frac{1}{c(\mathbf{x})^2 \rho(\mathbf{x})} \delta P_s(\mathbf{x},t) \; d\mathbf{x} \bigg|_{t=T}$$

$$+\int_0^T\int_\Omega \delta P_s(\mathbf{x},t)\frac{1}{c(\mathbf{x})^2\rho(\mathbf{x})}\frac{\partial^2 q(\mathbf{x},t)}{\partial t^2}\;d\mathbf{x}\,dt-\int_0^T\int_{\partial\Omega} q(\mathbf{x},t)\frac{1}{\rho(\mathbf{x})}\frac{\partial}{\partial n}\delta P_s(\mathbf{x},t)\,d\sigma\,dt$$

$$-\int_0^T\int_\Omega \delta P_s(\mathbf{x},t)\boldsymbol{\nabla}\cdot\left[\frac{1}{\rho(\mathbf{x})}\boldsymbol{\nabla}q(\mathbf{x},t)\right]\;d\mathbf{x}\,dt =$$

$$\int_0^T\int_\Omega \delta c(\mathbf{x}) \left[\frac{1}{c(\mathbf{x})^3 \rho(\mathbf{x})} \frac{\partial^2 P_s(\mathbf{x},t)}{\partial t^2}\right] q(\mathbf{x},t)\ d\mathbf{x}\, dt$$

$$-\int_0^T\int_\Omega \delta\rho(\mathbf{x}) \left[\frac{1}{c(\mathbf{x})^2\rho(\mathbf{x})^2}\frac{\partial^2 P_s(\mathbf{x},t)}{\partial t^2} - \frac{1}{\rho(\mathbf{x})^2}\boldsymbol{\nabla} P_s(\mathbf{x},t)\cdot\boldsymbol{\nabla}\right] q(\mathbf{x},t)\ d\mathbf{x}\,dt$$

$$+\int_0^T\int_{\partial\Omega}\delta\rho(\mathbf{x})\left[\frac{1}{\rho(\mathbf{x})^2}\frac{\partial P_s(\mathbf{x},t)}{\partial n}\right]q(\mathbf{x},t)\,d\sigma\,dt-\int_0^T\int_{\Omega}\delta f_s(t)\delta(\mathbf{x})q(\mathbf{x},t)\ d\mathbf{x}\,dt.$$

$t = 0$

$$\partial P/\partial n = \hat{\mathbf{n}} \cdot \boldsymbol{\nabla} P$$

$$\left\{ \frac{1}{c(\mathbf{x})^2 \rho(\mathbf{x})} \frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla} \cdot \left[\frac{1}{\rho(\mathbf{x})} \boldsymbol{\nabla} \right] \right\} q(\mathbf{x}, t)$$

$$= -\sum_{r,s} \frac{\partial}{\partial z} \left\{ \delta(\mathbf{x} - \mathbf{x}_r) \frac{w(t)}{\rho(\mathbf{x})} \left[D_{rs}(t) - \frac{1}{\rho(\mathbf{x})} \frac{\partial P_s(\mathbf{x}, t)}{\partial z} \right] \right\}$$

$t = T$

$$q(\mathbf{x},t)|_{t=T} = \left.\frac{\partial}{\partial t}q(\mathbf{x},t)\right|_{t=T} = q(\mathbf{x},t)|_{\partial\Omega} = 0.$$

δJ

$$\nabla_c J(\mathbf{x})$$

$$\frac{\delta J}{\delta c(\mathbf{x})} = \int_0^T \frac{1}{c(\mathbf{x})^3 \rho(\mathbf{x})} \frac{\partial^2 P_s(\mathbf{x}, t)}{\partial t^2} q(\mathbf{x}, t) dt.$$

$$\nabla_{\rho} J(\mathbf{x})$$

$$\frac{\delta J}{\delta \rho(\mathbf{x})} = \int_0^T \left[\frac{1}{c(\mathbf{x})^2 \rho(\mathbf{x})^2} \frac{\partial^2 P_s(\mathbf{x},t)}{\partial t^2} - \frac{1}{\rho(\mathbf{x})^2} \boldsymbol{\nabla} P_s(\mathbf{x},t) \cdot \boldsymbol{\nabla} \right] q(\mathbf{x},t) dt$$

$$-\sum_{r,s}\int_0^T \frac{\delta(\mathbf{x}-\mathbf{x}_r)}{\rho(\mathbf{x})^2}\left[D_{rs}(t)-\frac{1}{\rho(\mathbf{x})}\frac{\partial P_s(\mathbf{x},t)}{\partial z}\right]\frac{\partial P_s(\mathbf{x},t)}{\partial z}\; w(t)dt$$

$$\nabla_s J(t)$$

$$\frac{\delta J}{\delta f_s(t)} = q(\mathbf{x}, t)\big|_{\mathbf{x}=\mathbf{0}}$$

$$\frac{1}{c^3 \rho} \frac{\partial^2 P}{\partial t^2}$$

$$\nabla_{\rho} J_1$$

$$f_s(t) = \delta(t)$$