$$d(\mathbf{x}_i, \mathbf{x}^k, t)$$

t

$$\mathbf{x}_i$$

$$\mathbf{x}^k$$

$$\|\mathbf{x}_i - \mathbf{x}^k\|$$

$$\|\mathbf{x}_i + \mathbf{x}^k\|/2$$

Δt_i

Δt^k

$$d(\mathbf{x}_i, \mathbf{x}^k, t) \approx d(t - \Delta t_i - \Delta t^k) \quad \text{for} \quad t_1 < t < t_2.$$

$$c_{ij}^{kl}(\tau)$$

$$c_{ij}^{kl}(\tau) \equiv c_{ji}^{lk}(-\tau) \equiv \int_{t_1}^{t_2} d(\mathbf{x}_i, \mathbf{x}^k, t-\tau)\ d(\mathbf{x}_j, \mathbf{x}^l, t)\ dt.$$

$$\|\mathbf{x}_i - \mathbf{x}_j\| < x_{\max} \ , \ \|\mathbf{x}^k - \mathbf{x}^l\| < x^{\max} \quad \text{and} \quad \|(\mathbf{x}_i + \mathbf{x}^k)/2 - (\mathbf{x}_j + \mathbf{x}^l)/2\| < x^{\max} \ .$$

the lags $\tau_{ij}^{kl} \approx \Delta t_i - \Delta t_j + \Delta t^k - \Delta t^l$ will maximize $c_{ij}^{kl}(\tau_{ij}^{kl})$.

τ_{ij}^{kl}

$$\text{find } \Delta t_i \text{ and } \Delta t^k \text{ to minimize } \sum_{ijkl} (\tau_{ij}^{kl} - \Delta t_i + \Delta t_j - \Delta t^k + \Delta t^l)^2.$$

$$C_i^k(\tau) \equiv \sum_{jl} c_{ij}^{kl}(\tau).$$

$$C_i(\tau) \equiv \sum_{jkl} c_{ij}^{kl}(\tau) \quad \text{and} \quad C^k(\tau) \equiv \sum_{ijl} c_{ij}^{kl}(\tau)$$

$$C_{ij}(\tau) \equiv \sum_{kl} c_{ij}^{kl}(\tau) \quad \text{and} \quad C^{kl}(\tau) \equiv \sum_{ij} c_{ij}^{kl}(\tau).$$

$$|\tau| < \tau_{\max}$$

$\{\tau_i^k, \tau_i, \tau^k, \tau_{ij}, \tau^{kl}\}$ maximize $C_i^k(\tau_i^k), C_i(\tau_i),\ C^k(\tau^k),\ C_{ij}(\tau_{ij}),$ and $C^{kl}(\tau^{kl})$.

$$\min_{\Delta t_i \Delta t^k} \quad \sum_{i,k}(\tau_i^k - \Delta t_i - \Delta t^k)^2 + \sum_i(\tau_i - \Delta t_i)^2 + \sum_k(\tau^k - \Delta t^k)^2 +$$

$$\sum_{i,j}(\tau_{ij} - \Delta t_i + \Delta t_j)^2 + \sum_{k,l}(\tau^{kl} - \Delta t^k + \Delta t^l)^2$$

$$\max_{\Delta t_i \Delta t^k} \left[C_i^k(\Delta t_i + \Delta t^k) + C_i(\Delta t_i) + C^k(\Delta t^k) + C_{ij}(\Delta t_i + \Delta t_j) + C^{kl}(\Delta t^k + \Delta t^l) \right]$$

$$d(\mathbf{x}_i, \mathbf{x}^k, t) \Leftarrow d(\mathbf{x}_i, \mathbf{x}^k, t - \Delta t_i - \Delta t^k).$$

$$c_{ij}^{kl}(\tau) \Leftarrow c_{ij}^{kl}(\tau - \Delta t_i + \Delta t_j - \Delta t^k + \Delta t^l).$$