A convenient approximation of transverse isotropy for higher-order moveout, prestack time migration, and depth calibration

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Introduction

Processors use different RMS velocity models for three steps of time imaging: NMO, DMO, and poststack time migration. To perform prestack time migration in a single step, we must use a single velocity model. A single step avoids an extra stationary-phase approximation and should produce more accurate results. Nevertheless, results are usually worse with a single velocity model, unless different velocities are used for flat and dipping reflections [8]. Those velocities which best fit prestack normal moveouts over offset (flat reflections) do not best focus the tails of diffractions over midpoint (dipping reflections). Conventional processing hides this difference with inconsistent velocity models for prestack moveout analysis and poststack migration.

Occasionally, processors want to use a higher-order normal-moveout equation to flatten prestack reflections with long offsets. Conventional moveout analysis does a good job of fitting the difference in traveltime between near and far offsets, but a higher-order moveout can better flatten any residual bulge in the middle. Our parameterization of this normal moveout should be consistent with the model of anisotropy used in full prestack time imaging.

It is recognized that the kinematics of surface reflection seismic data are insensitive to component of transverse isotropy that is essential for an accurate conversion of time to depth. We can isolate the parameters needed to fit surface reflection times and allow depths to be calibrated independently.

A single convenient analytic approximation of transverse isotropy will allow us to generalize prestack moveout, time imaging, and depth conversion. No more parameters will be introduced than necessary. Parameters are decoupled so that each can be estimated in turn, if the additional degrees of freedom are required to fit the data.

Much of this material originally appeared in an appendix to Harlan [7].

Parameters for approximate transverse isotropy

Assume that anisotropic velocities have a vertical axis of symmetry, like the transversely isotropic (TI) media described in Thomsen [11]. Although that paper is titled "Weak elastic anisotropy," the same parameterizations can be applied to very strong anisotropy [12].

Three of Thomsen's parameters, V_z, δ, and ϵ, are defined by the elastic constants of a general TI medium. These constants can be used to specify three different effective velocities at a single point in the model. V_z is the velocity of a wave traveling vertically along the axis of symmetry. The velocity in any horizontal direction V_x is defined by

$$\epsilon = V_x^2(V_z^{-2} - V_x^{-2})/2, \tag{1}$$
$$V_x^2 = (1+2\epsilon)V_z^2, \text{ and} \tag{2}$$
$$V_x \approx (1+\epsilon)V_z. \tag{3}$$

A "normal moveout velocity" (NMO) velocity V_n is defined by

$$\delta \equiv V_n^2(V_z^{-2} - V_n^{-2})/2, \tag{4}$$
$$V_n^2 = (1+2\delta)V_z^2, \text{ and} \tag{5}$$
$$V_n \approx (1+\delta)V_z. \tag{6}$$

If these TI properties represent the equivalent medium of many isotropic layers [3, 10], then we can expect $\epsilon > \delta$ [4]. Using Backus averaging, Phil Anno of Conoco has also shown that we can expect $\epsilon > 0$ and $\delta < 0$, if the V_s/V_p ratio and V_s have a positive correlation. These inequalities imply that $V_n \leq V_z \leq V_x$. Well calibrations have shown that shales can produce $\delta > 0$. Such shales possess "intrinsic" anisotropy that cannot be modeled as the macroscopic equivalent of isotropic layers.

Researchers at the Colorado School of Mines [12, 2] have also defined a constant

$$\eta \equiv (\epsilon - \delta)/(1+2\delta) \tag{7}$$
$$= V_x^2(V_n^{-2} - V_x^{-2})/2, \tag{8}$$
$$V_x^2 = (1+2\eta)V_n^2, \text{ and} \tag{9}$$
$$V_x \approx (1+\eta)V_n. \tag{10}$$

For an equivalent medium of isotropic layers $\eta > 0$.

Many combinations of three of these parameters $V_z, V_x, V_n, \epsilon, \delta, \eta$ can be used to describe a TI medium for compressional P waves with a known axis of symmetry. Such an approximation has already dropped a fourth constant (shear wave velocity) to which compressional waves are insensitive.

The parameter η is an excellent parameter choice for maximum sensitivity to the kinematics of surface measurements only. If TI velocities are parameterized by V_x, η, and δ, we find that surface measurements are very insensitive to δ. We could not make the same claim if ϵ were used instead of η.

I prefer the three velocities V_z, V_x, and V_n because they share the same units. Surface measurements are very insensitive to V_z, given values for V_x and V_n. This choice is not the most convenient for processing, however.

Phase and group velocities

The exact equations for TI phase velocity as a function of angle are rather clumsy, and *no explicit form* is available for group velocity. Explicit approximate equations can fit the same family of curves almost as well as the original correct equations [9]. I use an approximate equation for group velocity which appears to emulate closely the exact curves for large ranges of positive ϵ and negative δ. Estimated curves usually have larger statistical errors from noisy data than introduced by these approximations.

Kirchhoff migrations can calculate traveltimes by integrating the group velocity along a stationary Fermat raypath. Fourier-domain implementations can use only the equation for phase velocities. Some raytracing methods use both, because phases must be matched across discontinuous boundaries.

I choose approximate curves with the three velocities V_z, V_x, and V_n. Let ϕ be the group angle of a raypath from the vertical. Then the group velocity $V(\phi)$ can be expressed as

$$V(\phi)^{-2} = V_z^{-2}\cos^2\phi + (V_n^{-2} - V_x^{-2})\cos^2\phi\sin^2\phi + V_x^{-2}\sin^2\phi, \quad (11)$$

$$V(\phi)^{-1} = V_x^{-1}\sqrt{1 + 2\eta\cos^2\phi\sin^2\phi + 2\epsilon\cos^2\phi} \quad (12)$$

$$\approx V_x^{-1}(1 + \eta\cos^2\phi\sin^2\phi + \epsilon\cos^2\phi) \quad (13)$$

$$\approx V_x^{-1}[1 + \eta\cos^2\phi(1 + \sin^2\phi) + \delta\cos^2\phi]. \quad (14)$$

Compare Byun et al [5], who use the same approximation with different parameters.

Greg Lazear of Conoco found that a symmetric equation approximates the phase velocity $v(\theta)$ as a function of the phase angle θ, but with reciprocals of the same velocity parameters:

$$v(\theta)^2 = V_z^2\cos^2\theta + (V_n^2 - V_x^2)\cos^2\theta\sin^2\theta + V_x^2\sin^2\theta. \quad (15)$$

Compare this phase equation closely to the group equation (11). I know of no other approximation that allows such symmetry.

Extended stacking moveouts

The normal-moveout (NMO) velocity V_n has a physical interpretation to justify its name. Imagine an experiment on a homogeneous and anisotropic medium, or imagine a small scale experiment on a smooth model. Measure the traveltime t_0 between two points placed on a vertical line, separated by a vertical distance $z = V_z t_0$. Now displace the upper point a distance h along a horizontal line (a normal-moveout) and measure the new traveltime t_h.

Then according to equation (11) the traveltime t_h as a function of offset h is exactly

$$t_h^2 = t_0^2 + \left[V_n^{-2} + (V_x^{-2} - V_n^{-2}) \frac{h^2}{h^2 + V_z^2 t_0^2} \right] h^2. \tag{16}$$

For small offsets $h \ll V_z t_0$, the value of t_h in this "moveout equation" is dominated by the NMO velocity V_n rather than V_x. For large offsets $h \gg V_z t_0$, the raypath is almost horizontal and V_x dominates.

I find it convenient to define a stacking velocity $V_h(h)$ as a function of the offset h for a fixed vertical distance $z = V_z t_0$:

$$\begin{aligned} V_h(h)^{-2} &\equiv (t_h^2 - t_0^2)/h^2 & (17) \\ &= V_n^{-2} + (V_x^{-2} - V_n^{-2}) \frac{h^2}{h^2 + V_z^2 t_0^2} & (18) \\ &= V_n^{-2} \left(1 - \frac{2\eta}{1 + 2\eta} \cdot \frac{h^2}{h^2 + V_z^2 t_0^2} \right). & (19) \end{aligned}$$

I use the term stacking velocity because I want to suggest the best-fitting curve over a finite range of offsets, as you would prefer for a stacking or semblance analysis.

Simplify the moveout equation (16) to fit a pseudo-hyperbola:

$$t_h^2 = t_0^2 + h^2 / V_h(h)^2. \tag{20}$$

The stacking velocity covers the range $V_n \leq V_h(h) \leq V_x$ for a Backus equivalent medium with negative δ, increasing in value as h increases. When $\eta = 0$, the curve is exactly hyperbolic, and $V_n = V_x$. Notice that this stacking velocity can measure a local property as well as an average to the surface. To use two-way reflection times in (20) we need only replace the half offset h by the full offset.

Three measurements of traveltimes at three different offsets h should uniquely determine the three velocity constants V_z, V_x, V_n. The traveltimes are much more sensitive to V_n, which determines moveouts at small offsets, and to V_x, which determines moveout at larger offsets. The vertical velocity V_z affects only the rate at which the stacking velocity (18) changes from one limit to the other. As long as V_z has roughly the correct magnitude, then we can fit all measured traveltimes very well.

For imaging surface data in time, we acknowledge our insensitivity to V_z and can approximate it with another value. We can approximate $V_z \approx V_n$ and simplify stacking velocity (19) even further, as in

$$V_h(h)^{-2} \approx V_n^{-2} \left(1 - 2\eta \frac{h^2}{h^2 + V_n^2 t_0^2}\right). \tag{21}$$

This new equation depends only on two parameters, V_x and V_n. A better approximation might be $V_z \approx V_n^2/V_x$, or equivalently $\epsilon \approx 2\delta$, which is probably closer to commonly observed values. NMO equation (16) now is equivalent to

$$t_h^2 \approx t_0^2 + \left[V_n^{-2} + (V_x^{-2} - V_n^{-2}) \frac{h^2}{h^2 + V_n^4 V_x^{-2} t_0^2}\right] h^2. \tag{22}$$

which is equivalent to equation (5) in Alkhalifah [1] and equation (7) in Grechka and Tsvankin [6]. Both these publications derive from Tsvankin and Thomsen [12], which uses an asymptotic correction of a Taylor expansion to arrive at this approximation.

The equivalent stacking velocity is then

$$V_h(h)^{-2} \approx V_n^{-2} \left(1 - 2\eta \frac{h^2}{h^2 + V_n^4 V_x^{-2} t_0^2}\right). \tag{23}$$

The differences between these two approximate stacking velocities (23) and (21) are negligible for numerical work. I will use the simpler version (21).

Moveout analyses determine the stacking velocity for a specific aperture of offsets. Define our best *isotropic* approximation of the velocity to be the stacking velocity V_{iso} at the maximum offset $h_{\max}$ of the aperture:

$$V_{\text{iso}} \equiv V_h(h_{\max}) \tag{24}$$

$$\approx V_n \left(1 + \eta \frac{h_{\max}^2}{h_{\max}^2 + V_n^2 t_0^2}\right). \tag{25}$$

Or we can assume that we know the aperture angle α from the vertical, so that

$$V_{\text{iso}} \approx V_n(1 + \eta \sin^2 \alpha), \text{ where } \tan\alpha = h/V_z t_0. \tag{26}$$

The best isotropic velocity V_{iso} is a simple function of infinitesimal-offset NMO velocity V_n and the anisotropic parameter η. Similarly we can use the definition of η in (8) to rewrite the above equation (26) as

$$V_{\text{iso}} \approx V_x(1 - \eta \cos^2 \alpha). \tag{27}$$

This form will prove to be very useful when rewriting our group velocity equations (13) and (14).

A conventional velocity analysis produces densely picked values for $V_{\rm iso}$. The anisotropic parameter η adjusts the moveout between near and far offsets. For anisotropic moveout analysis, we could use the following offset-dependent stacking velocity to scan for η, holding $V_{\rm iso}$ constant:

$$V_h(h)^{-2} \approx V_{\rm iso}^{-2} \left[1 + 2\eta \left(\frac{h_{\rm max}^2}{h_{\rm max}^2 + V_{\rm iso}^2 t_0^2} - \frac{h^2}{h^2 + V_{\rm iso}^2 t_0^2} \right)\right]. \tag{28}$$

Conventional moveout analysis is not very sensitive to the anisotropic parameter η except for unusually wide-aperture data, with offsets greater than depth. Much more anisotropic information is available by performing a full prestack time migration.

For prestack time migration, we can expect that conventional analysis for $V_{\rm iso}$ will best describe the moveouts of flat reflections. Dipping reflections are difficult to pick in prestack semblance analysis because they are sparser and move across midpoints, from gather to gather, as migration velocity changes.

Holding $V_{\rm iso}$ constant, we can perturb η at a low spatial resolution until the imaging of steep reflections improves. As η changes, there will be a small adjustment of the bulge in flat reflections over offset, with little effect on a fully time-migrated stack. The moveouts of dipping reflections, on the other hand, will change drastically as η changes, with swings from positive to negative residual moveouts, and with lateral movement over midpoint. As η improves, you should see fault-plane reflections sharpen and focus in targeted prestack time-migrated images Flat reflections should change little, and $V_{\rm iso}$ should require little revision after updating η. By contrast, an optimization of V_n and η requires both to be adjusted simultaneously, with equal resolution.

Adjustment for depth ties

The group velocity equation (13) is useful for Kirchhoff depth imaging. We can focus images very well with good values for V_x and V_n, (or $V_{\rm iso}$ and η), then adjust imaged depths to tie wells with V_z (or ϵ, or δ) while holding the other two parameters constant.

If we have used tomographic methods to estimate isotropic *interval* velocities, then we should attempt also to estimate the effective aperture angle α at each depth in the model.

To recapituate our algebra, we combine the definitions of η (9) and of stacking velocity $V_h(h)$ (19) to solve for the horizontal velocity:

$$V_x^{-1} \approx V_h(h_{\rm max})^{-1} \left(1 - \eta \frac{V_z^2 t_0^2}{h_{\rm max}^2 + V_z^2 t_0^2}\right) \tag{29}$$

$$\approx V_{\rm iso}^{-1}(1 - \eta \cos^2 \alpha). \tag{30}$$

At maximum offset, we recover our previous relationship (27) between $V_{\rm iso}$ and V_x. Substitute this horizontal velocity into the group velocity (13). We can then adjust the isotropic velocities $V_{\rm iso}$ with η and ϵ according to

$$V(\phi)^{-1} \approx V_{\rm iso}^{-1}(1 - \eta \cos^2 \alpha)(1 + \eta \cos^2 \phi \sin^2 \phi + \epsilon \cos^2 \phi). \quad (31)$$

The isotropic velocity $V_{\rm iso}$ best explains the moveouts and traveltimes of relatively flat reflections over the finite aperture. The parameter η modifies these velocities at high dips, to image steep reflections better without degrading the imaging of low-dip reflections. The third parameter ϵ has little effect on measured surface traveltimes at any dip (holding $V_{\rm iso}$ and η constant), but can be adjusted as necessary to tie wells. We can also use layered medium theory to predict this ϵ from estimated η and V_x. Or if shale dominates, with strong intrinsic anisotropy and $\delta > 0$, then correlations can be calibrated for a given area. At worst, we know $0 < \epsilon$, so we can assume a default value of $\eta/2 < \epsilon < 2\eta$, as appropriate for a given area. Such a default value is still better than a default value of 0.

These three parameters $V_{\rm iso}$, η, and ϵ do not completely decouple the steps of anisotropic velocity analysis, but they should minimize the number of iterations necessary for revisions.

If you prefer to use δ instead of ϵ as the third degree of freedom, then simply use the definition of η in (7) for

$$V(\phi)^{-1} \approx V_{\rm iso}^{-1}(1 - \eta \cos^2 \alpha)[1 + \eta \cos^2 \phi(1 + \sin^2 \phi) + \delta \cos^2 \phi]. \quad (32)$$

Adjustment of narrow-aperture velocities

Although I greatly prefer the approach in the preceding section, many prefer to treat their estimated isotropic velocities as equivalent to NMO velocities V_n. Such an assumption is not a bad one if angles are limited during interval velocity estimation. Dix inversion of stacking velocities may be closer to V_n if stacking velocities were consciously optimized for inner offsets. The Common Reflecting Surface tomography of Karlsrühe University inverts only the curvature of reflection traveltimes around zero-offsets.

For such approaches, one might prefer a different triplet of velocity parameters: V_n, η, and δ. Group velocity can be described by replacing V_x in the approximation (14) with V_n and η as in equation (10). Additionally we can replace ϵ with δ, using the definition of η in (7):

$$V(\phi)^{-1} \approx V_n^{-1}(1 - \eta \sin^4 \phi + \delta \cos^2 \phi) \quad (33)$$

$$\approx V_n^{-1}[1 - \eta(1 - \cos^2 \phi \sin^2 \phi) + \epsilon \cos^2 \phi]. \quad (34)$$

Again V_n and η should be sufficient to model all surface reflection traveltimes, for all dips. Holding these two constant, we can adjust either δ or ϵ to tie known well depths.

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