Programming for Geophysics

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WHILE YOU WERE DRONING I SLAMMED OUT SOME BETA CODE AND PUT IT ON THE INTERNET FOR COMMENTS.

Some ways to program as a geophysicist

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- Build a personal library. Generalize your code for reuse.

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- Do not get carried away.

Learn best software practices

- Show and share
- Source control
- Tests
- Small changes (refactoring)
- Appropriate generalization

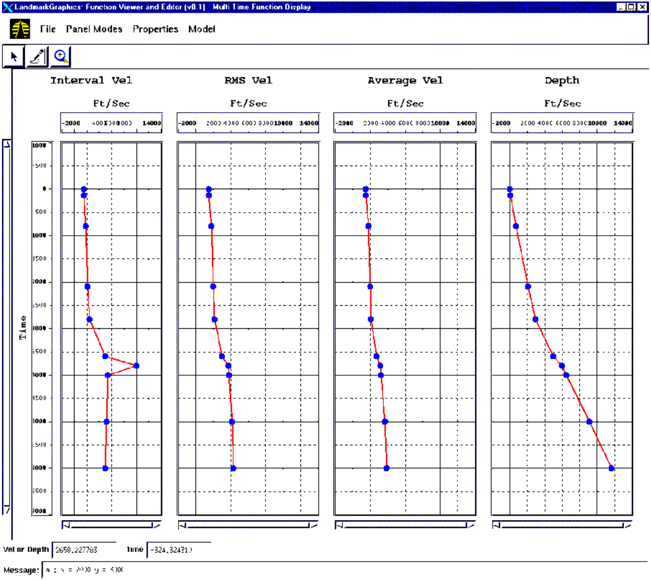
Examples of generalization/abstraction

- Seismic data objects with flexible dimensions
- Separate velocity models from ray tracers
- Different imaging conditions with different extrapolators

Typical geophysical inversions

- Data simulated by series of non-linear operations
- Inversion is both over- and under-determined
- No model parameters fit data perfectly
- Many models fit data equally well
- Non-linearity is well-behaved

Sensitivity of interval velocity to RMS errors



Dix inversion

- Forward equation cannot fit arbitrary data:

$$V_j^{\mathrm{rms}} = \sqrt{\frac{1}{j}\sum_{k=1}^{j}(V_k^{\mathrm{int}})^2}$$

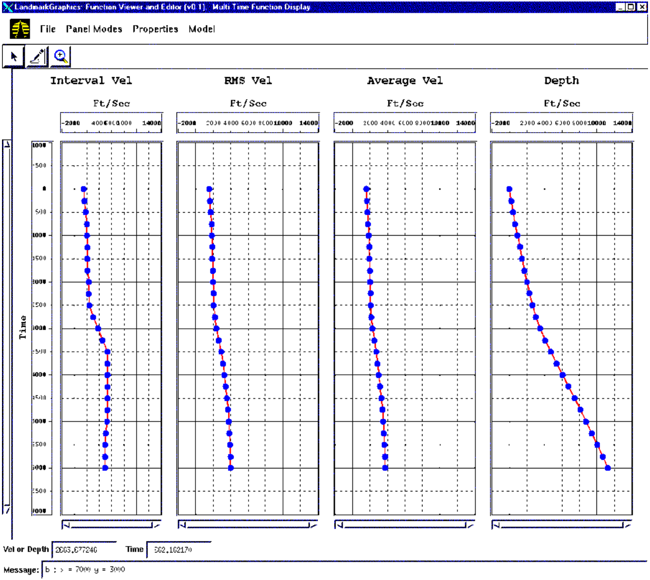
- Explicit inverse may not be physical:

$$V_j^{\mathrm{int}} = \sqrt{j(V_j^{\mathrm{rms}})^2 - (j-1)(V_{j-1}^{\mathrm{rms}})^2}$$

- Instead minimize damped least-squares:

$$\sum_j \left\{ (V_j^{\mathrm{rms}})^{-2} - \left[\frac{1}{j}\sum_{k=1}^{j}(V_k^{\mathrm{int}})^2\right]^{-1} \right\}^2 + \epsilon \sum_k (V_k^{\mathrm{int}})^{-2}$$

Damp interval velocity roughness



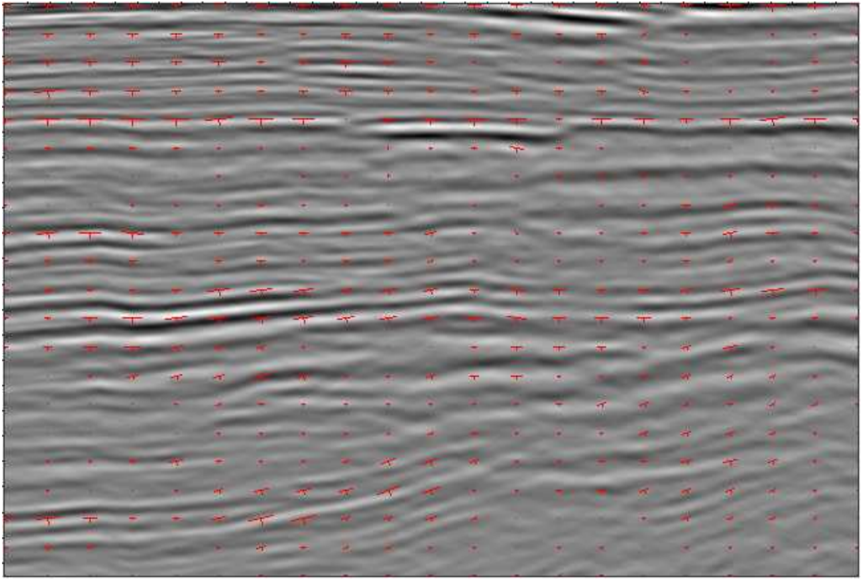
Defining an inversion

- Do not define your solution by the way you solve it.
- Want to improve the solution without redefining the problem.
- Instead, identify an objective function (or probabilities).
  E.g., define rays by minimum time.

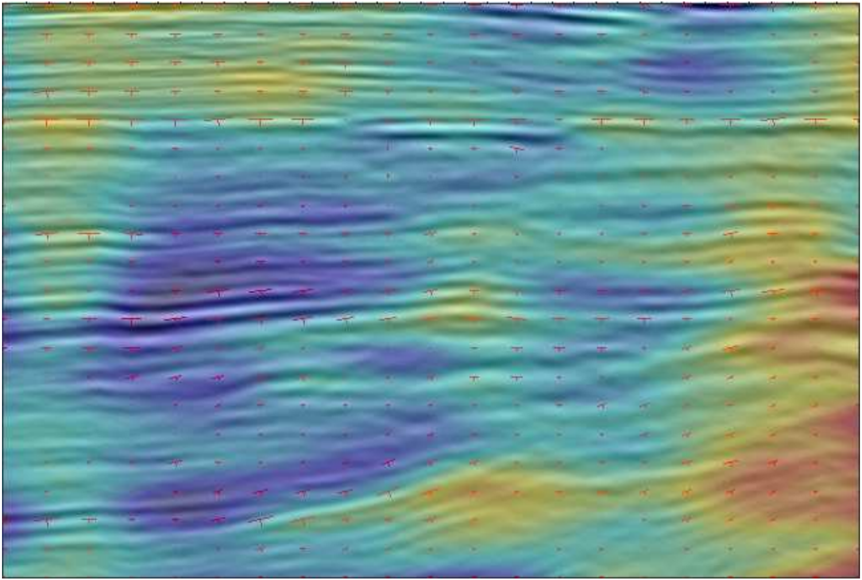
Lomask's flattening, redone

- Estimate vertical stretch that flattens reflections.
- Original: Custom regression, phase-unwrapping
- New version: A few hundred lines of code specific to inversion
- JTK reused: structure tensors, Gaussian filters, Gauss-Newton

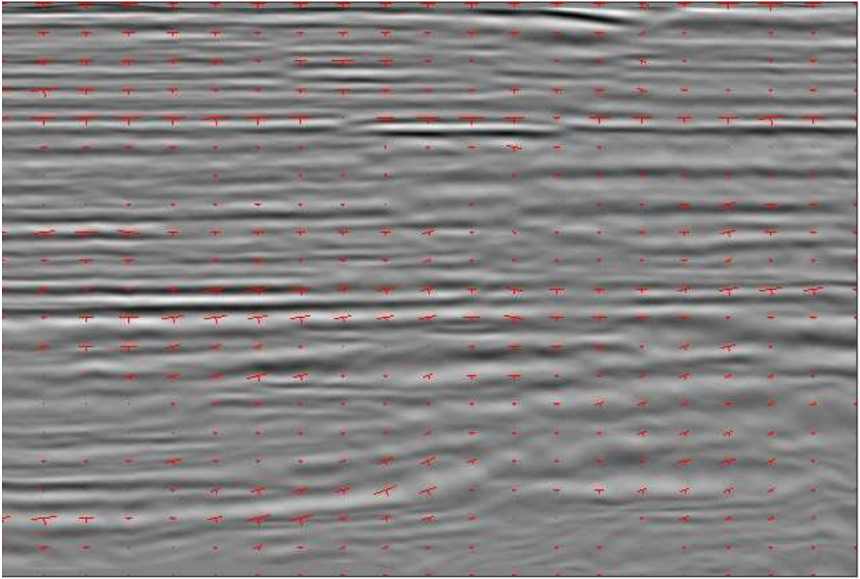
Local dips from structure tensors



Estimated vertical shifts in color



Flattened with vertical shifts



The problem, the data, and the solution

Flatten seismic structure with vertical shift $\tau(x, y, t)$:

$$\text{flat}(x, y, t) = \text{structure}[x, y, t + \tau(x, y, t)].$$

Data are slopes p_x, p_y measured from structure tensors.

$$\begin{aligned} \text{Want} \quad \frac{\partial}{\partial x}\tau(x, y, t) &\approx p_x(x, y, t) \\ \text{and} \quad \frac{\partial}{\partial y}\tau(x, y, t) &\approx p_y(x, y, t). \end{aligned}$$

$$\begin{aligned} \min_{\tau(x,y,t)} \iiint \Big(&\left\| \frac{\partial}{\partial x}\tau(x, y, t) - p_x(x, y, t) \right\|^2 \\ &+ \left\| \frac{\partial}{\partial y}\tau(x, y, t) - p_y(x, y, t) \right\|^2 \\ &+ \epsilon \left\| \tau(x, y, t) \right\|^2 \Big) \, dx \, dy \, dt \end{aligned}$$

Looks like damped least-squares

The best model $\mathbf{m}$ fits the data $\mathbf{d}$ with a function $\mathbf{f}(\mathbf{d})$ by minimizing the vector norms

$$\|\mathbf{d} - \mathbf{f}(\mathbf{m})\|_d^2 + \|\mathbf{m}\|_m^2$$

$$\text{or} \quad [\mathbf{d} - \mathbf{f}(\mathbf{m})]^* \, \underset{\sim}{\mathbf{C}}_d^{-1} \, [\mathbf{d} - \mathbf{f}(\mathbf{m})] + \mathbf{m}^* \underset{\sim}{\mathbf{C}}_m^{-1} \mathbf{m}.$$

Optional covariances:

$$\underset{\sim}{\mathbf{C}}_d \equiv E(\mathbf{d}\,\mathbf{d}^*) \text{ and } \underset{\sim}{\mathbf{C}}_m \equiv E(\mathbf{m}\,\mathbf{m}^*).$$

Gauss-Newton inversion

- Finds $\mathbf{m}$ to minimize

$$[\mathbf{d} - \mathbf{f}(\mathbf{m})]^* \underset{\sim}{\mathbf{C}}_d^{-1} [\mathbf{d} - \mathbf{f}(\mathbf{m})] \; + \; [\mathbf{m} - \mathbf{m}_0]^* \underset{\sim}{\mathbf{C}}_m^{-1} [\mathbf{m} - \mathbf{m}_0]$$

- Algorithm:
  1. Accepts starting model $\mathbf{m}_0$
  2. Approximates $\mathbf{f}(\mathbf{m}_0 + \Delta\mathbf{m}) \approx \mathbf{f}(\mathbf{m}_0) + \underset{\sim}{\mathbf{F}} \cdot \Delta\mathbf{m}$
  3. Conjugate-gradients minimizes quadratic for $\Delta\mathbf{m}$
  4. Line search scales perturbation: $\mathbf{m}_0 + \alpha\Delta\mathbf{m}$
  5. Adds perturbation to reference model for new $\mathbf{m}_0$
  6. Returns to step 2

Required operations

- Simulate data from model:
  $\mathbf{d} = \mathbf{f}(\mathbf{m})$
- Perturb data with model perturbation:
  $\Delta\mathbf{d} = \underset{\sim}{\mathbf{F}}(\mathbf{m}_0) \cdot \Delta\mathbf{m} \; \approx \; \mathbf{f}(\mathbf{m}_0 + \Delta\mathbf{m}) - \mathbf{f}(\mathbf{m}_0)$
- Perturb model with transpose:
  $\underset{\sim}{\mathbf{F}}(\mathbf{m}_0)^* \cdot \Delta\mathbf{d}$

What is that transpose?

Use definition: $\mathbf{d}^*(\underset{\sim}{\mathbf{F}}\mathbf{m}) \equiv (\underset{\sim}{\mathbf{F}}^*\mathbf{d})^*\mathbf{m}$

- Discrete: swap summations
- Continuous: integrate by parts

Examples:

- Smoothing $\rightarrow$ Smoothing
- Convolution $\rightarrow$ Correlation
- Derivative $\rightarrow$ Negative derivative
- Plane-wave modeling $\rightarrow$ Slant stacks
- Seismic modeling $\rightarrow$ Migration

Inversion sees three abstract operations

```
Vector data  = forward(Vector model)
Vector data  = linearized(Vector model, Vector refModel)
Vector model = transpose(Vector data, Vector refModel)
```

Required operations for both data and model

```
Vector {
  scale(float scalar)           [required]
  add(Vector other)
  dot(Vector other)

  multiplyInverseCovariance()  [optional]
  applyHardConstraint()
}
```

Constrained Dix inversion

- Solve for smooth interval slowness **m**.
- Minimize errors in squared stacking slowness **d**
- Forward transform:

$$1/d_j = (1/j) \sum_{k=1}^{j} (1/m_k^2)$$

- Linearized transform:

$$\Delta d_j = (2\, d_j^2/j) \sum_{k=1}^{j} (1/m_k^3) \Delta m_k$$

- Transpose transform:

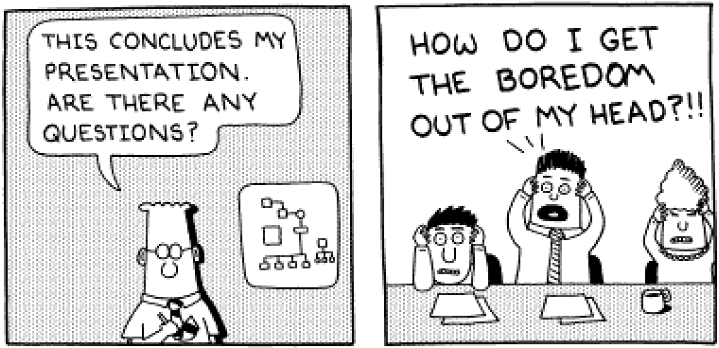
$$\Delta m_k = (1/m_k^3) \sum_{j=k}^{\infty} (2\, d_j^2/j) \Delta d_j$$

Other applications

- Tomography: reflection, cross-well, diving, amplitude
- Generalized Radon transforms
- Surface-consistent deconvolution
- Normal moveout corrections
- Automatic moveout picking
- Coherency, wavelet/phase attributes
- Tests for simulations

Conclusions

- More time on “computer science” quickly saves time
- Look for opportunities to generalize


THIS CONCLUDES MY PRESENTATION. ARE THERE ANY QUESTIONS?
HOW DO I GET THE BOREDOM OUT OF MY HEAD?!!

Alternative to covariance

- Insert simplification filter $\mathbf{d} = \mathbf{f}(\underset{\sim}{\mathbf{S}} \cdot \mathbf{m})$ where $\underset{\sim}{\mathbf{S}}^* \underset{\sim}{\mathbf{C}}_m^{-1} \underset{\sim}{\mathbf{S}} \approx \underset{\sim}{\mathbf{I}}$
- If $\underset{\sim}{\mathbf{C}}_m^{-1} \cdot \mathbf{m}$ roughens, then $\underset{\sim}{\mathbf{S}} \cdot \mathbf{m}$ smooths.
- Faster than covariance constraint
- Can change dynamically during optimization